Running-time Analysis of Broadcast Consensus Protocols

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Introduction

The Setting

- distributed computation

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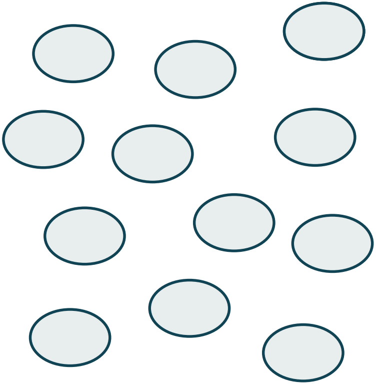

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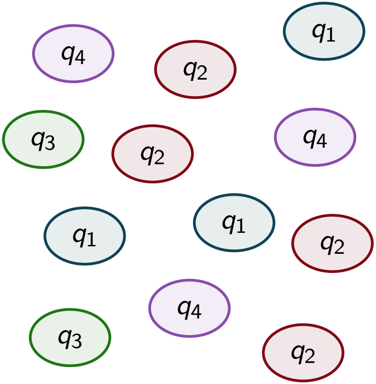

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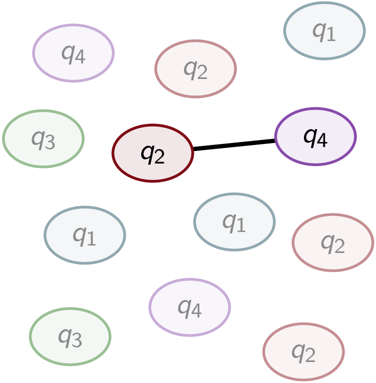

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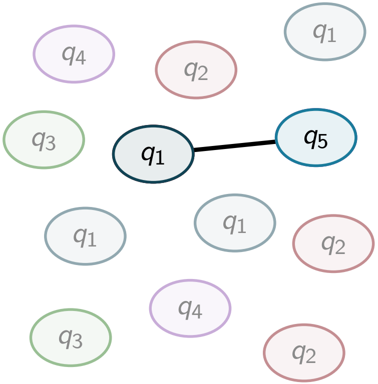

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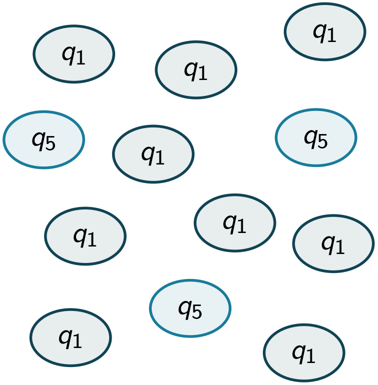
- random interactions
- want to reach consensus on whether the initial configuration satisfies a property



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  - $\mathcal{O}(n^{1+\varepsilon})$ interactions to converge [Kosowski, Uznański 2018]

Broadcasts Consensus Protocols

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4. Construct faster and more powerful protocols

Results

Prior work:

- Blondin, Esparza and Jaax show that BCPs compute exactly NL
  - no bounds on running time
  - multiple stages of reduction $\rightarrow$ complicated protocols

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1. time-optimal[1], simple protocols for semi-linear predicates
   - expected $\mathcal{O}(n \log n)$ transitions

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Our results:

1. time-optimal[1], simple protocols for semi-linear predicates
   - expected $\mathcal{O}(n \log n)$ transitions
2. poly-time BCPs are precisely ZLP
   - i.e. predicates decidable by zero-error, log-space, expected poly-time randomised Turing Machines

[1] w.r.t. number of transitions

What is a Broadcast Consensus Protocol (BCP)?

$$\text{BCP} = \text{Population Protocol} + \text{Broadcasts}$$

Formally:

finite set of states Q,
transitions $B : Q \to Q \times Q^Q$

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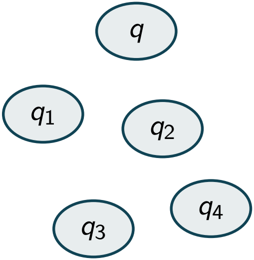
- Pairwise interactions can be simulated
- Non-determinism can be simulated

Transitions

Run on population of agents $C \in \mathbb{N}^Q$ (multiset of states).



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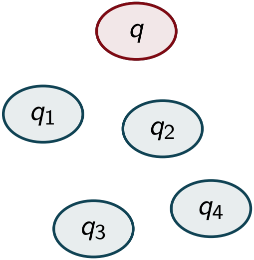

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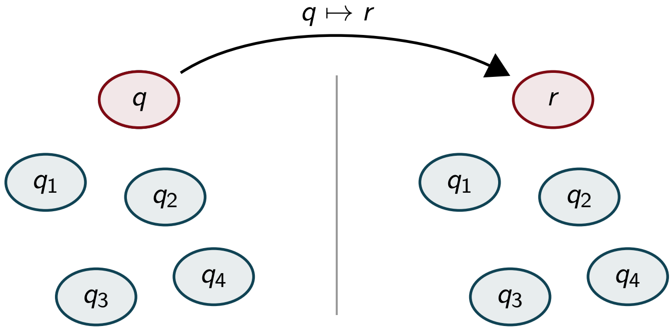

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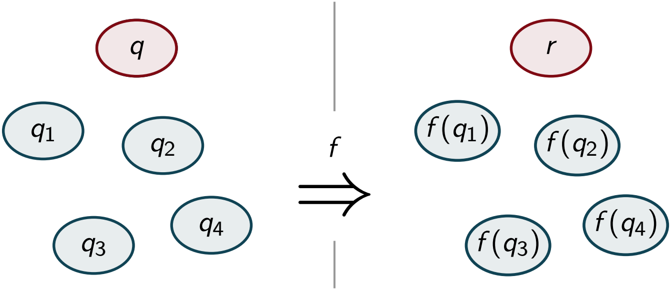

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Computation

initial states $I \subseteq Q$
output mapping $O : Q \to \{0, 1\}$
predicate $\varphi : \mathbb{N}^I \to \{0, 1\}$

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output mapping $O : Q \to \{0, 1\}$
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How do we compute φ ?
Pick agents at random until everyone has (and retains) the same output.

Example

Majority $\varphi(x, y) \Leftrightarrow x \geq y$

$(x, y) = (2, 3)$

input

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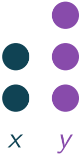

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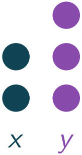

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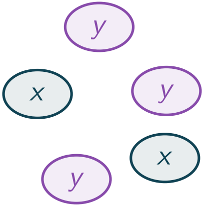
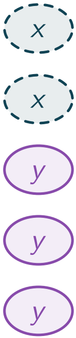


population

Compute $\varphi(x, y) \Leftrightarrow x \geq y$



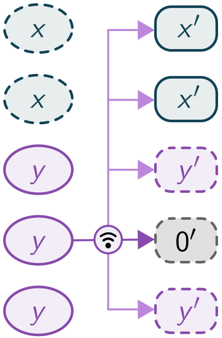
output 1

output 0

“active”

“disabled”

Compute $\varphi(x, y) \Leftrightarrow x \geq y$ $\qquad$ $y \mapsto 0, \{x \mapsto x', y \mapsto y', 0 \mapsto 0'\}$



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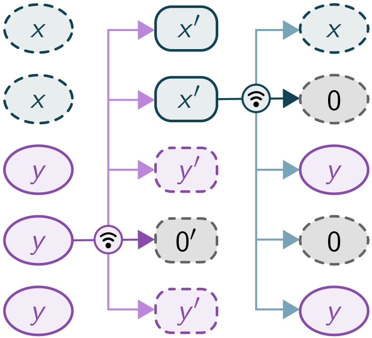

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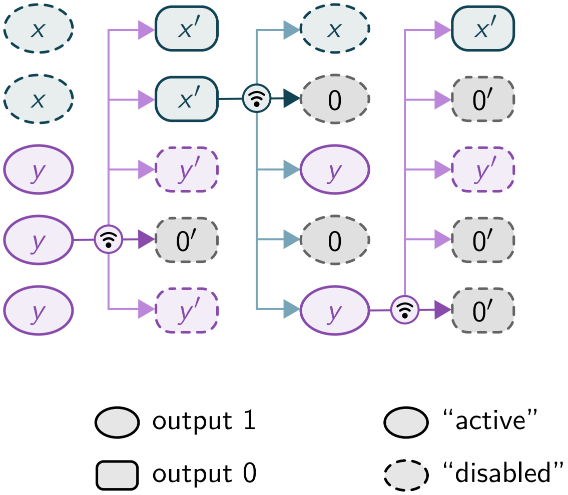

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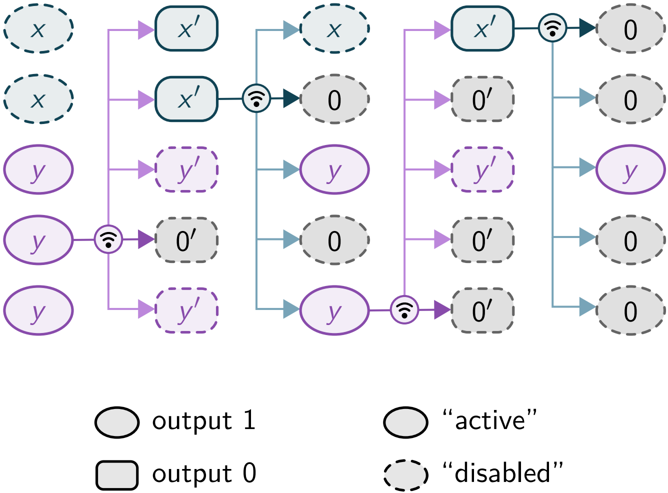

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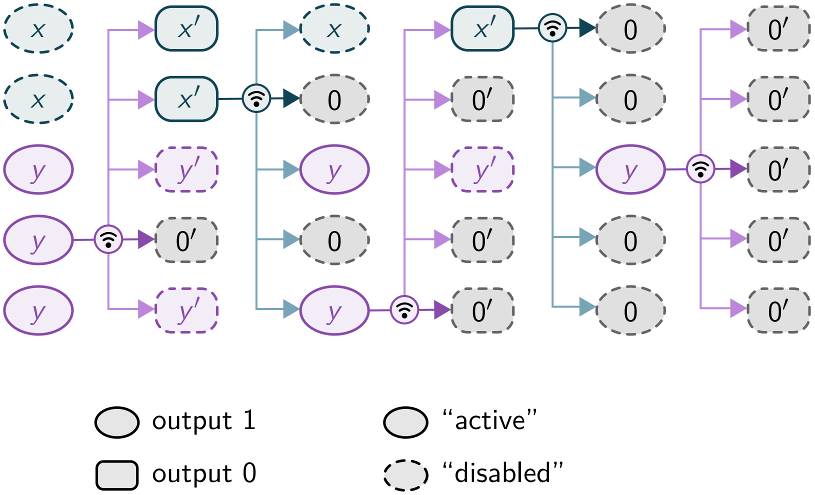

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Steps:

1. Decompose semi-linear predicate into boolean combination of modulo and threshold predicates
2. Protocol for modulo predicates
3. Protocol for threshold predicates
4. Boolean combinations (simple)

Modulo predicates

$$a_1 x_1 + ... + a_l x_l \equiv b \pmod{k}$$

Global state is $\{0, ..., k-1\}$, additions modulo k

Threshold predicates

$$a_1 x_1 + ... + a_l x_l \geq k$$

Global state is large enough counter, take care not to overflow.

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Thus we get time-optimal BCPs for semi-linear predicates.

Thank you for
your attention!